Worst Case Orbit Shadow Analysis

The worst case shadowing for a circular orbit is when the line Earth Sun line lies in the orbit plane. In all other orientations, the time spent in the shadow of the Earth on each orbit will be smaller than that shown in the plots below. Let the radius of the Earth be denoted by R_{Earth} , the radius of the circular orbit under consideration be denoted by R_{orbit} , and let μ denote the gravitational parameter of the Earth. Assume that the shadow of the Earth is a cylinder of radius R_{Earth} which stretches behind the Earth. This assumption is quite good out past geosynchronous orbit, and the times in shadow resulting from it are, if anything, a bit longer than the actual times in shadow. Figure 1, below, shows the geometry.

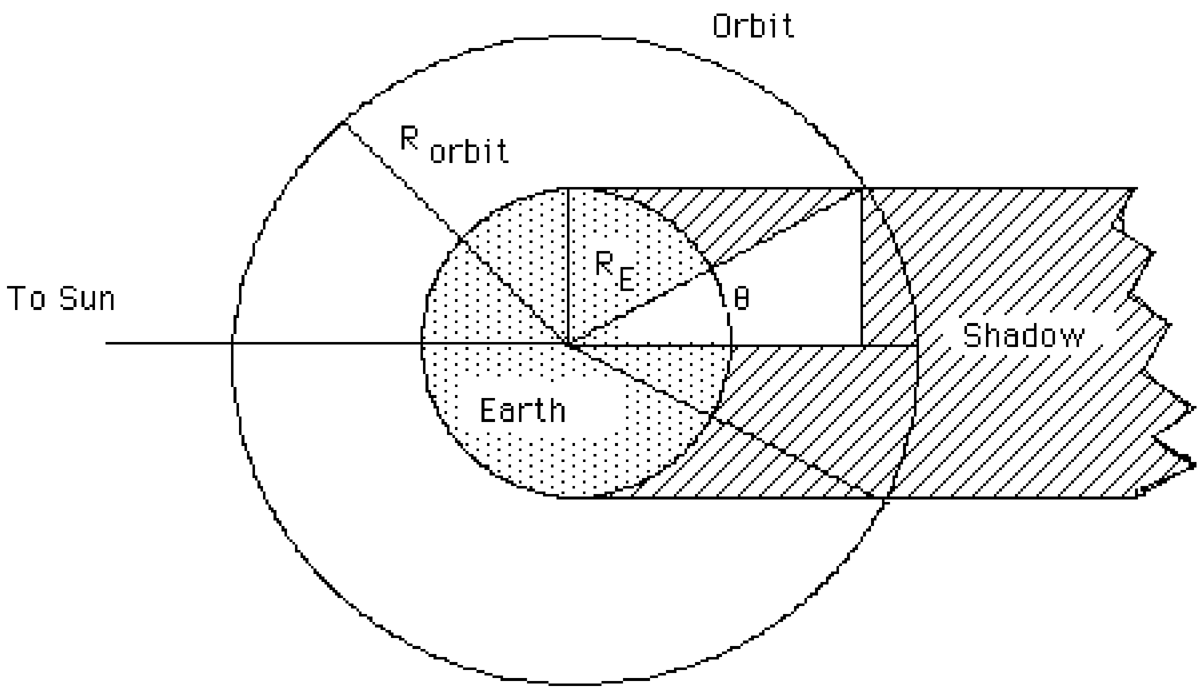

Figure 1: Earth's Shadow

Given an orbit of radius R_{orbit}, the angle θ (the half angle of the shadow) can be detetermined from $\sin(\theta) = R_{Earth} / R_{orbit}$. The total angle during which the spacecraft is in the Earth's shadow is 2θ , and the fraction of the orbit that the spacecraft is in the Earth's shadow is $f_{shadow} = \frac{2\theta}{2\pi} = \frac{\theta}{\pi}$. The period, T_{orbit} ,of the orbit is given by $T_{orbit} = 2\pi\sqrt{\frac{R^3_{orbit}}{\mu}}$, and the time, t_{shadow} , spent by the spacecraft in Earth's shadow on each orbit is given by $t_{shadow} = f_{shadow} T_{orbit}$.

Plot 2, below, shows the fraction of the circular orbit spent in Earth's shadow as a function of the radius of the orbit.

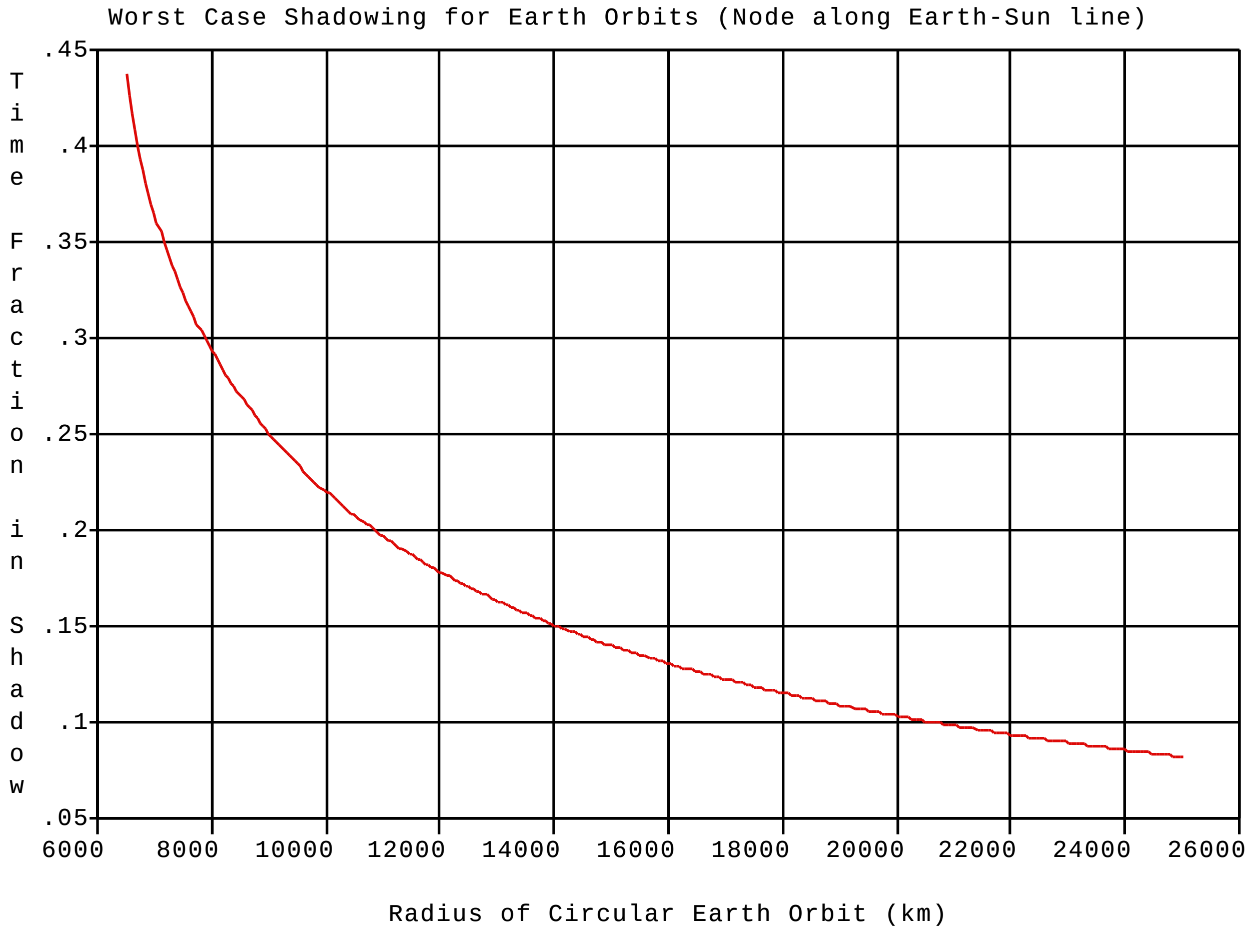

Figure 2. Fraction of a Circular Earth Orbit Spent in Earth's Shadow (Worst Case)

For an orbit just above the surface of the Earth, the fraction of the orbit spent in shadow is 50%. The fraction of the orbit spent in shadow goes down sharply as the orbital radius increases. Under the assumption of a cylindrical shadow, a circular orbit at geosynchronous radius which contains the Earth-Sun line will be in shadow only 4.8% of the time, and the duration in the shadow is 69.4 minutes. Under the same worst case assumptions, a satellite at lunar distance is in the shadow only 0.53% of the time (just over half of one percent) but the duration in shadow is over 208.6 minutes.

Figure 3, below, shows the orbital period and the time in shadow for circular Earth orbits under the worst case situation where the orbit plane contains the Earth-sun line.

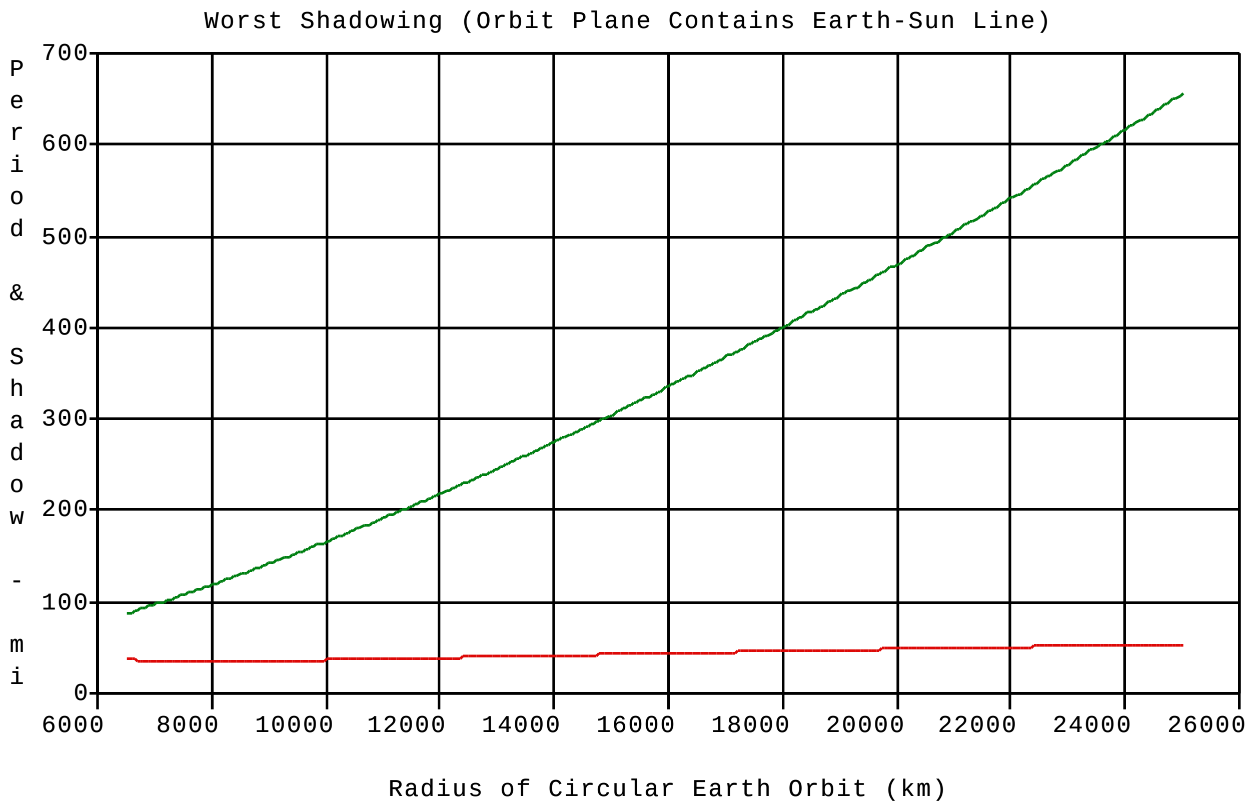

Figure 3. Orbit Period and Time in Shadow for Circular Earth Orbit (Worst Case)

In Figure 3, the top curve is the period of the circular orbit at the radius shown on the horizontal axis while the lower curve shows the time spent in Earth shadow per orbit by a spacecraft in that orbit. Note that over the range of orbital radii shown, the time spent in shadow varies very little. Under the assumption of a cylindrical shadow, a circular orbit at geosynchronous radius which contains the Earth-Sun line will be in shadow for 69.4 minutes. Under the same worst case assumptions, a satellite at lunar distance is in the shadow for about 208.6 minutes.

Note that the shadow duration curve in Figure 3 seems to have a very slight minimum in the region where the orbital raduis is about 8000 km. Figure 4, on the next page, explores this region by plotting the shadow duration alone without the orbital period curve. This greatly expands the vertical scale of the plot.

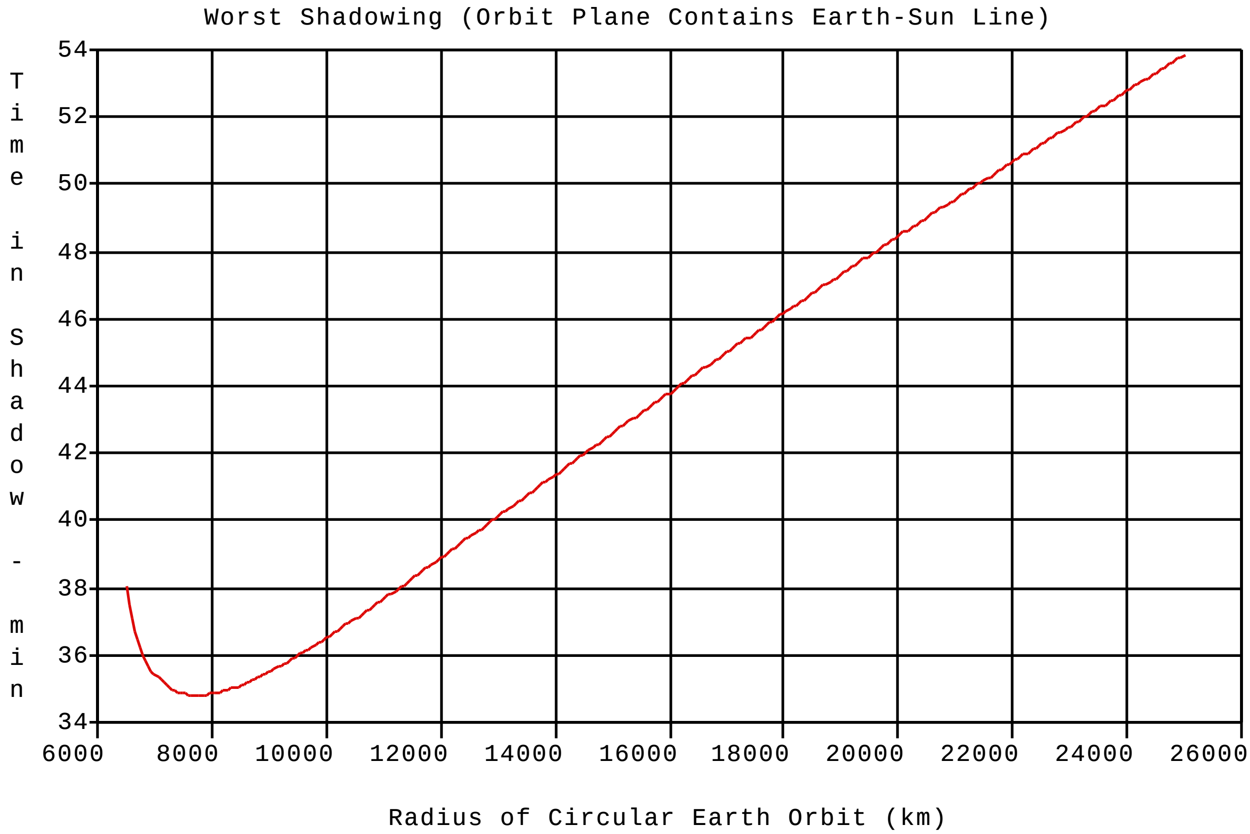

Figure 4. Time in Shadow for Circular Earth Orbit (Worst Case)

Note that with the expanded vertical scale, the time period per orbit that the spacecraft spends in the shadow under worst case conditions is readily apparent. Also note that there is a minimum in the curve where r is just under 8000 km. The actual minimum occurs at an orbital radius of 7743 km and the corresponding time in shadow is just under 35 minutes (34 min 48.4 sec).