Sun Synchronous Orbits
for the
Earth Solar Power Satellite System

Some of the most promising orbits for the Earth Solar Power System are circular Sun synchronous orbits which never enter Earth's shadow. In these orbits, gravity gradient stabilized "power towers" will orbit in planes which are within about 30° of perpendicular to the Earth-Sun line and the solar arrays can be set for near optimum power capture. The solar synchronicity of the orbits is caused by the Earth's oblateness (i.e., the J_2 term in Earth's geopotential).

There are a number of quantities which must be considered together when choosing orbits for the Earth Solar Power System. However, the analysis is quite simple. If we choose circular Sun-synchronous orbits as our baseline configuration, the choice of the inclination of the orbit determines the orbital altitude (radius & period) as well as the displacement in longitude between successive ascending or descending nodes (and hence the distance between successive ascending or descending nodes at the equator). The choice of inclination also determines the maximum latitude excursions away from the poles made by the groundtrack of satellites in the orbit.

Plots 1 through 5 have been developed with respect to circular Sun synchronous Earth orbits. Figures 1 and 2 show the variation in orbital radius and orbital altitude versus orbital inclination, respectively, for circular Sun synchronous orbits. Figures 3 and 4 show the angular and linear distances, respectively, between successive ascending (descending) nodes measured along the equator of the Earth versus orbital inclination for circular Sun synchronous orbits. Figure 5 is a plot of the orbital periods of circular Sun synchronous orbits versus orbital inclination.

In order for an orbit to be Sun synchronous, the node line must move eastward at a rate of one revolution per year. This motion, caused by the oblateness of the Earth, is described by the equation

$$\omega_{J_2} = -\frac{3}{2} * (\mathrm{Re}\,q / p)^2 n\ J_2 * \cos(i)$$

where $p = a\left(1-e^2\right)$, $e = 0$, $J_2 = .0010826$, $\mathrm{Re}\,q = 6378.16\ km$, and i is the inclination of the orbit. The rotation rate of the Earth is given by $\omega_{earth} = .000072921\ rad/s$.

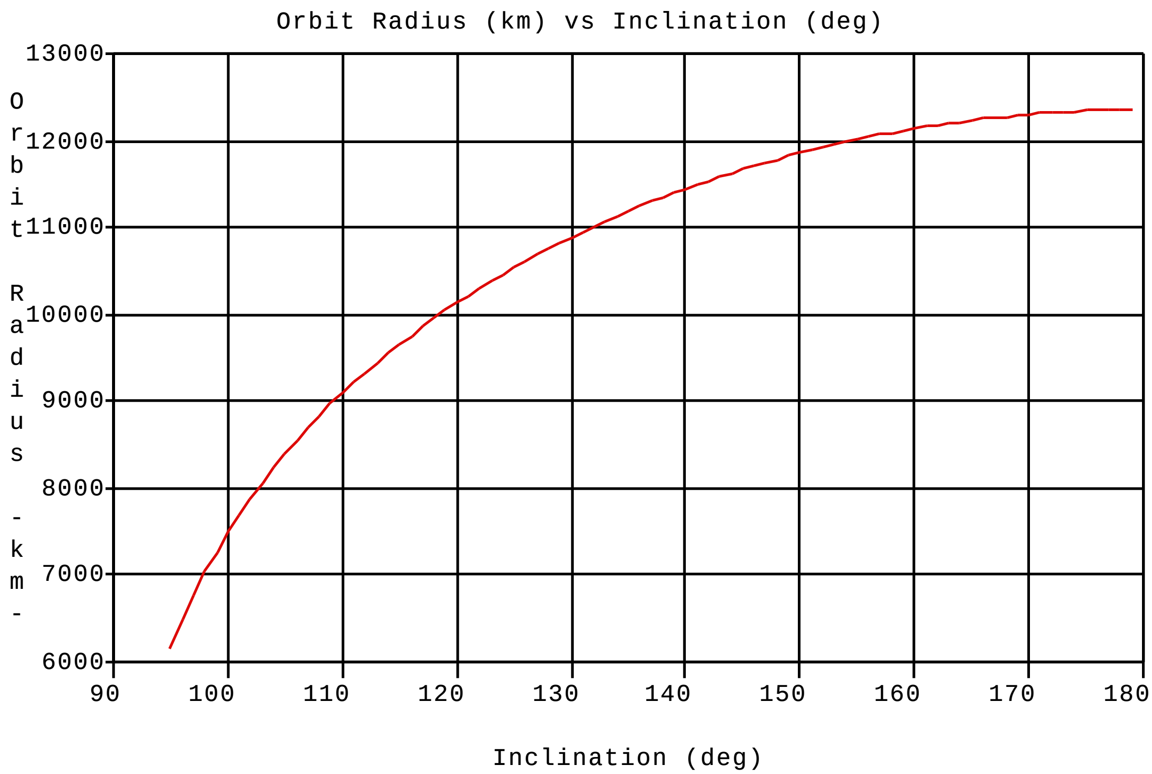


Figure 1. Orbital Radius versus Orbital Inclination for Circular Sun Synchronous Earth Orbits

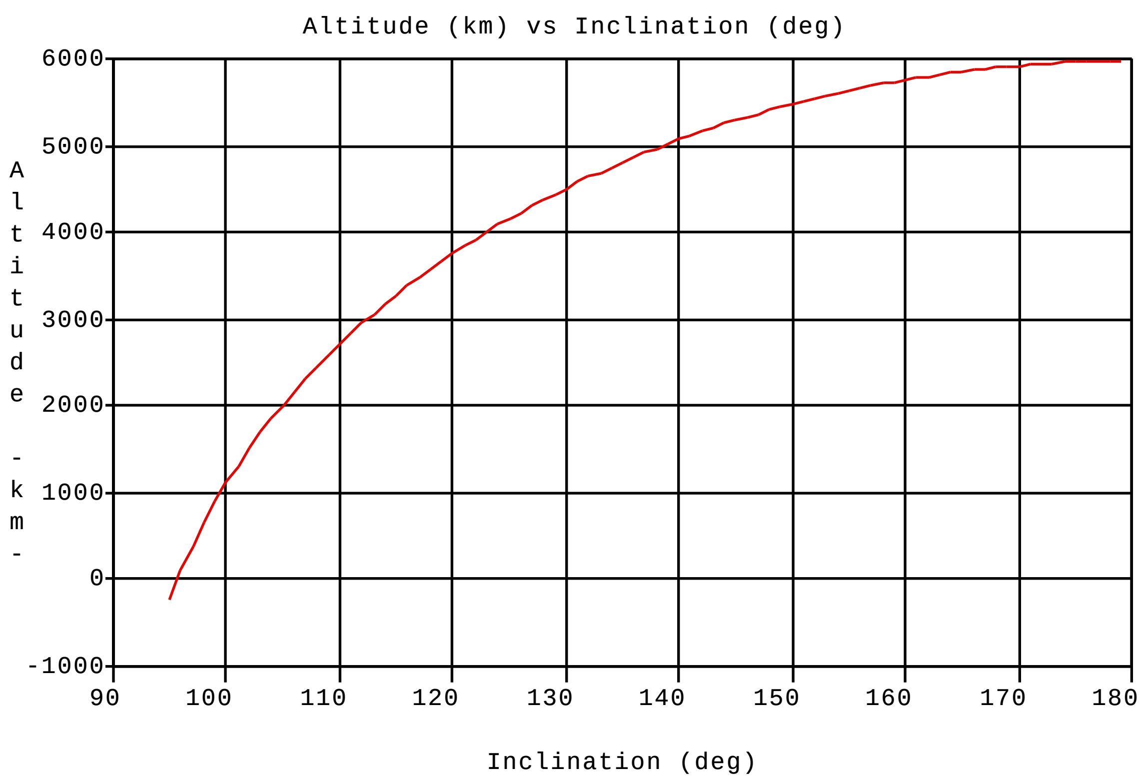


Figure 2. Orbital Altitude versus Orbital Inclination for Circular Sun Synchronous Earth Orbits

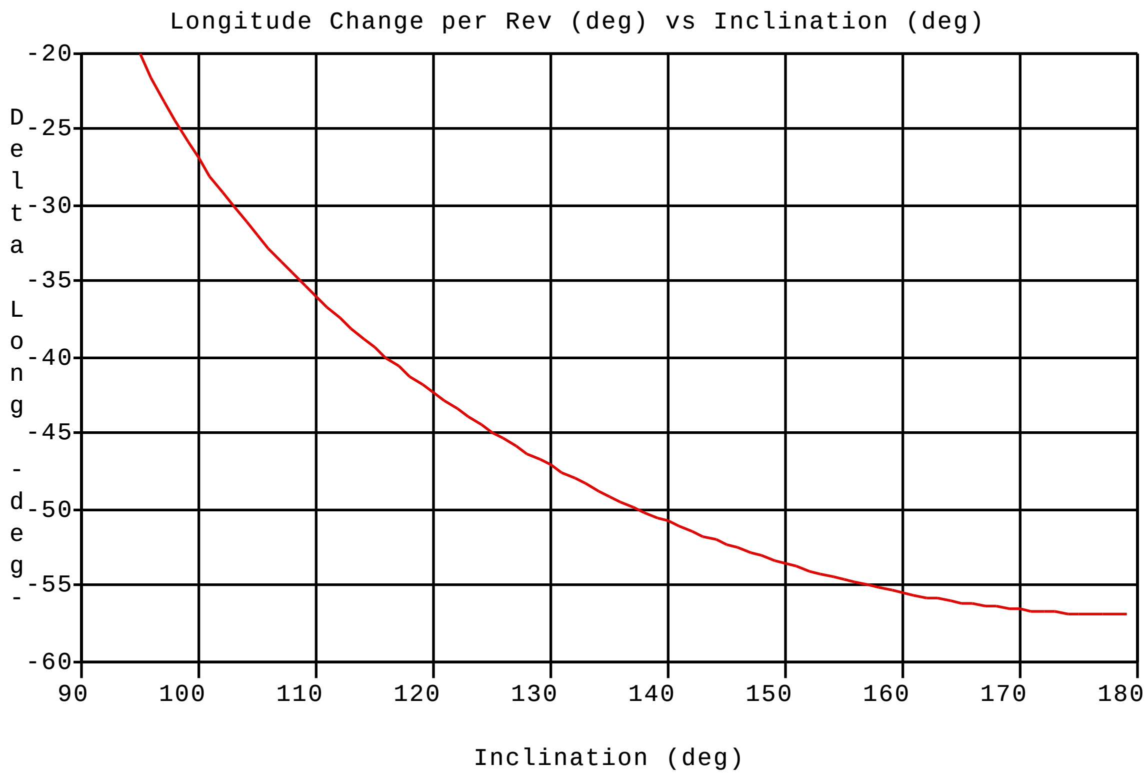


Figure 3. Angular Displacement per Revolution (Measured Along the Equator Relative to a Rotating Earth) versus Orbital Inclination for Circular Sun Synchronous Earth Orbits

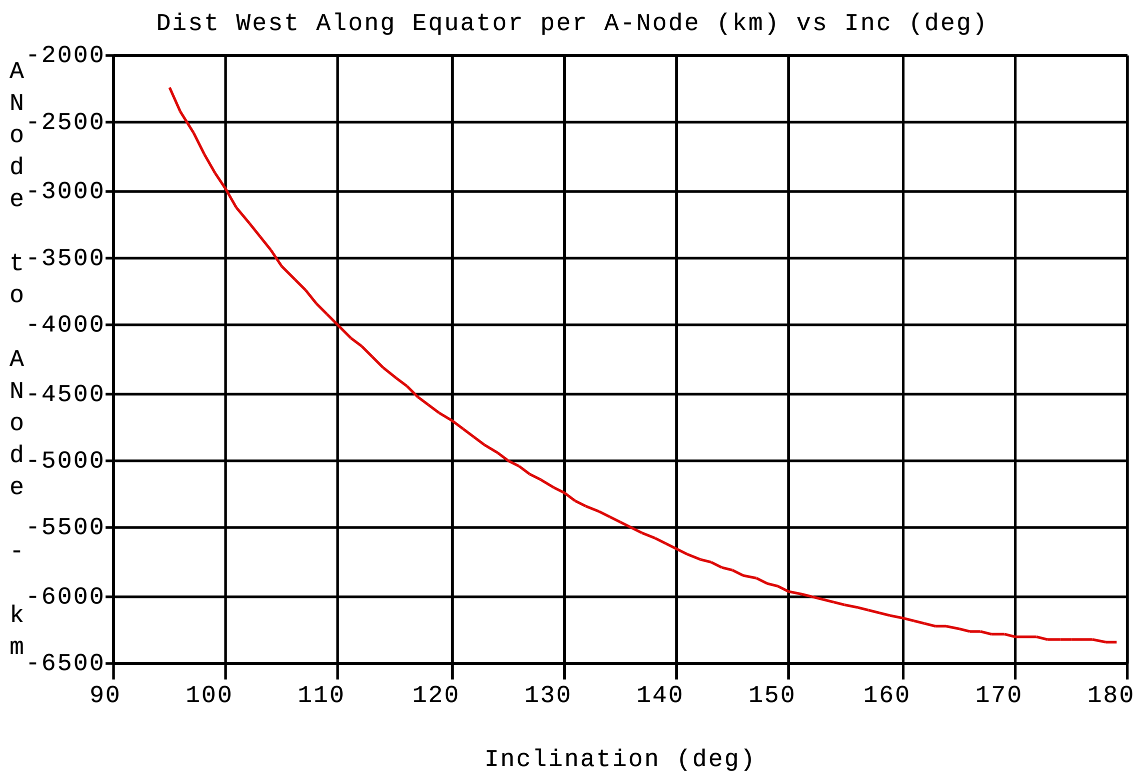


Figure 4. Linear Displacement per revolution Measured Along the Equator versus Orbital Inclination for Circular Sun Synchronous Earth Orbits

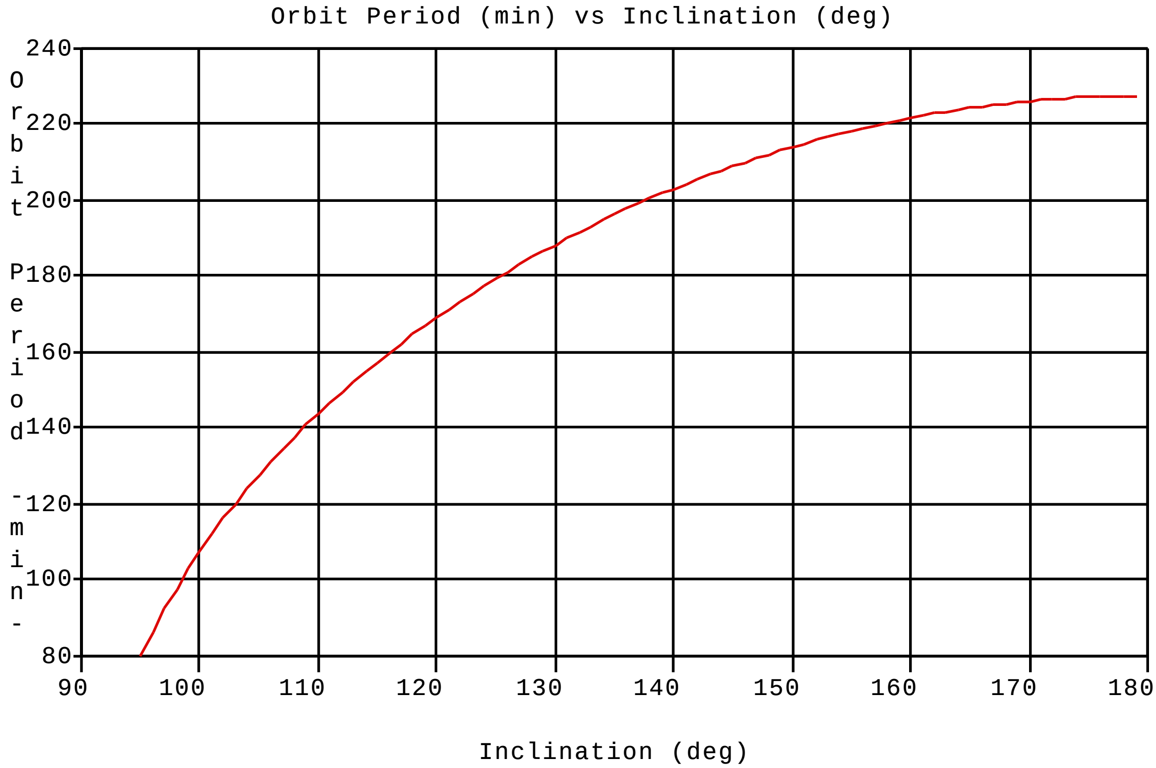


Figure 5. Orbital Period versus Orbital Inclination for Circular Sun Synchronous Earth Orbits

The following facts, which have been developed from interpretation of these plots, are noted.

1. All Sun synchronous orbits are retrograde (i.e., they have inclinations in the range $90° \leq i \leq 180°$.

2. The latitude range covered by the groundtrack of a Sun synchronous orbit is $\pm \Delta \delta$ where $| \delta | = (180° - i)$. This means, for example, that in order for the ground track to go 60° north and south of the equator, the inclination of the Sun synchronous orbit would be 120°, or if the groundtrack were to go 70° north and south of the equator, the inclination of the orbit would be 110°.

3. Assuming that the orbital altitudes lower that 1000 km would produce too much drag, the minimum inclination available for Sun sy;nchronous orbits will be about 100°. Lower inclinations, which are closer to polar orbits, would lead to Sun synchronous orbits at altitudes below 1000 km. Sun synchronous orbits having inclinations between 90° and 96° are impossible because the require orbital altitudes are less than the radius of the Earth.

4. Sun synchronous orbits with inclinations in the range $150° \leq i \leq 180°$ are unacceptable because their ground tracks do not go far enough north or south. For $| \delta | = 30°$. A range such that $| \delta | \geq 50°$ is more realistic from the point of view of geographic coverage for power transmission to the surface of the Earth. As a point of reference, Anchorage, Alaska is located at a latitude of about 60° North.

5. The closer a circular Sun synchronous orbit is to polar, the lower the orbit altitude.

6. Sun synchronous orbits with ground tracks which reach from 50° to 80° north and south (inclinations between 100° and 130°) will have altitudes lying in the range between 1000 km and 4500 km.

7. Sun synchronous orbits with inclinations between 100° and 130° have periods in the range of 108 minutes to just over 188 minutes.

8. Sun synchronous orbits with inclinations between 100° and 130° have ascending nodes (relative to the Earth) which are 27° to 47° apart, The corresponding distances between ascending nodes (measured along the Earth's equator) are between 3000 km and 5240km.

Additional Geometric Considerations

The angle between the orbit plane of a Sun synchronous orbit and the Earth-Sun line will not remain constant. Consider the case of an orbit for which, on the date of the winter solstice (approximately Dec 21st), the angle between the orbital plane and the Earth-Sun line, β called the Sun angle, is 90°. On this date, the orbital angular momentum vector of the orbit points directly away from the Sun. This orbit will have an inclination of approximately 113.5°. The satellite's ascending node will be ahead of the Earth along the Earth's velocity vector around the Sun. The line of nodes lies in the plane perpendicular to the Earth-Sun line.

Three months later, at the time of the vernal equinox (about March 21st), the inclination of the orbit will remain the same and the node line will have precessed 90°. The ascending node will still be "ahead" of the Earth but will not be on the velocity vector since the Earth's velocity vector will not, at this time, lie in the equatorial plane. The satellite's orbital angular momentum vector will have precessed such that the line of nodes still lies in the plane perpendicular to the Earth-Sun line, but the northern part of the orbit plane is inclined toward the Sun. The angle between the orbital plane and the Earth-Sun line is now 90° - 23.5° (~ 66.5°) . Note that the inclination of the orbit with respect to Earth's equator has not changed, but the line of nodes has precessed 90° during these three months.

At the time of the summer solstice, the orbit will again be in a plane such that the line of nodes is in the plane perpendicular to the Earth-Sun line. However, the angle between the orbital plane and the Earth-Sun line will now be about 43° (90° - 2*23.5°). Again, the northern half of the orbit will be inclined toward the Sun.

Earth Shadow Effects

Figure 6 is a plot of the raduis of the shadow of the Earth, measured perpendicular to the Earth-Sun line and Rsmallest, the minimum component of a circular Sun synchronous orbit normal to the Earth-Sun line versus orbital inclination.

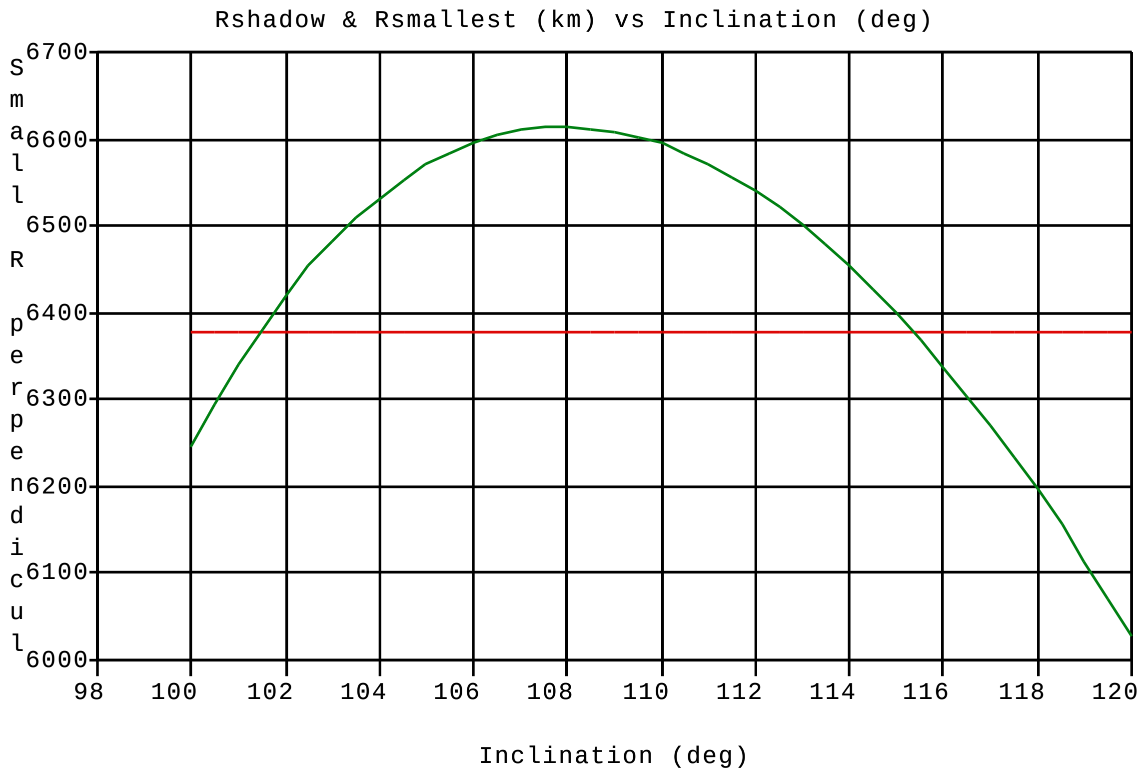


Figure 6. Rshadow (the radius of Earth's Shadow) and Rsmallest (the Minimum Circular Sun Synchronous Orbit Component Normal to the Earth-Sun Line) versus Orbital Inclination.

The Earth / Sun / orbit geometry for Sun Synchronous orbits with angular momentum vectors that project onto the Earth - Sun line (±) is such that orbits with inclinations greater than 115.36° go into the Earth's shadow at least once per year. Sun synchronous orbits with angular momentum vectors which project onto the ecliptic plane at a an angle ø away from the Earth Sun line go into the Earth's shadow at inclinations even nearer to polar.

Thus, the inclinations for Sun synchronous orbits which do not go into the shadow of the Earth must be less than 115.36° (radii ˘ 9694 km or altitudes ≤ 3316 km). The periods of these orbits are less than 9500 seconds (158 1/3 minutes). The angular displacement per revolution relative to the rotating Earth for such orbits is ≤ 39.6°, measured on the equator (39.6° is equivalent to 4407 km between successive northbound equator crossings).

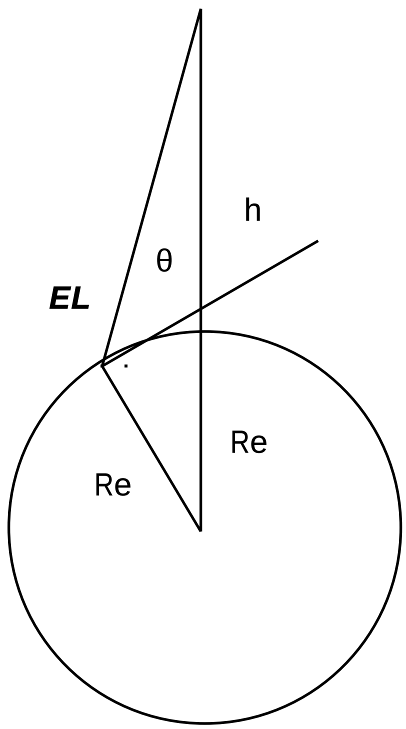
Also, note from Figure 6 that for circular Sun synchronous orbits with inclinations less than about 101.5° (101.45° is closer), the satellite again goes into the shadow of the Earth, even with the line of nodes along the 6AM - 6PM line. Thus, if we only use orbits that remain in constant sunlight, our orbits are restricted to

101.45° < Inclination < 115.36°
7781 km < Radius < 9694 km
1403 km < Altitude < 3316 km
113.9 min < Period < 158.3 min
28.5° < Nodal Displacement* per rev < 39.6°
3170 km < Nodal Displacement* per rev < 4407 km

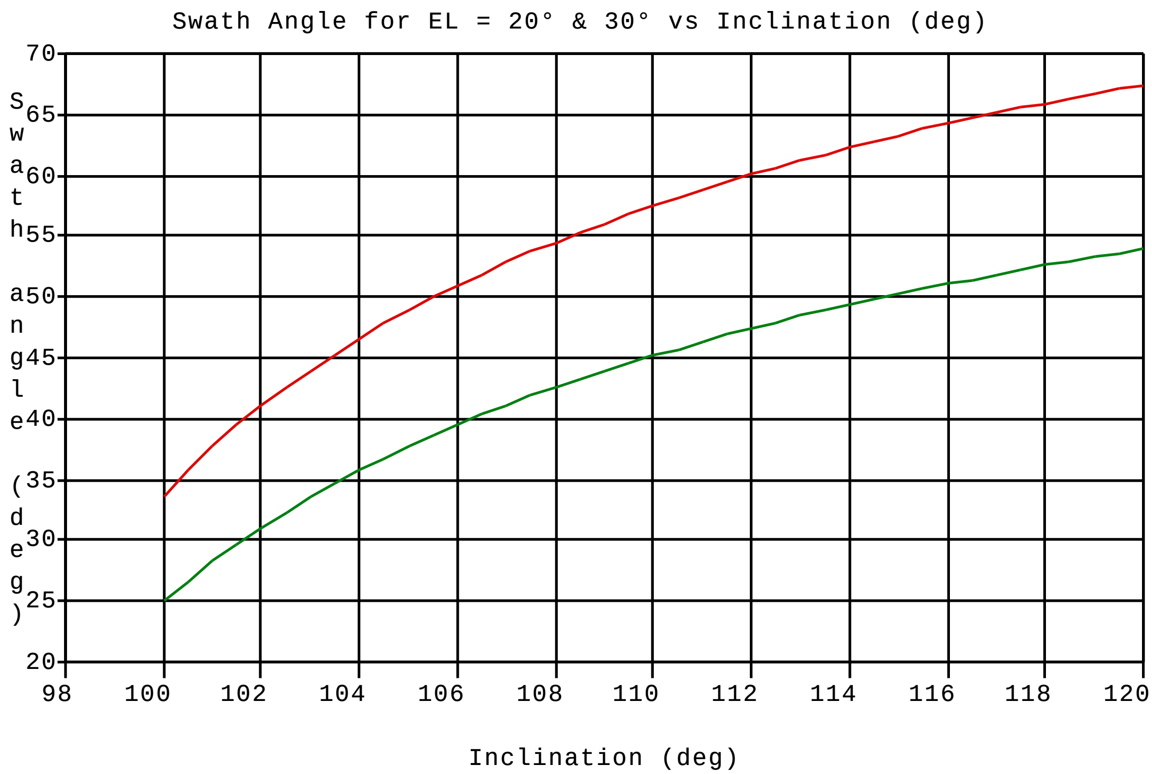
* Nodal displacements are measured relative to the surface of the Earth. They are dominated by the effect of the rotation of the Earth (about 15° per hour), but also include the precession due to J2 (one revolution per year). These two effects are in opposite directions for Sun synchronous orbits.

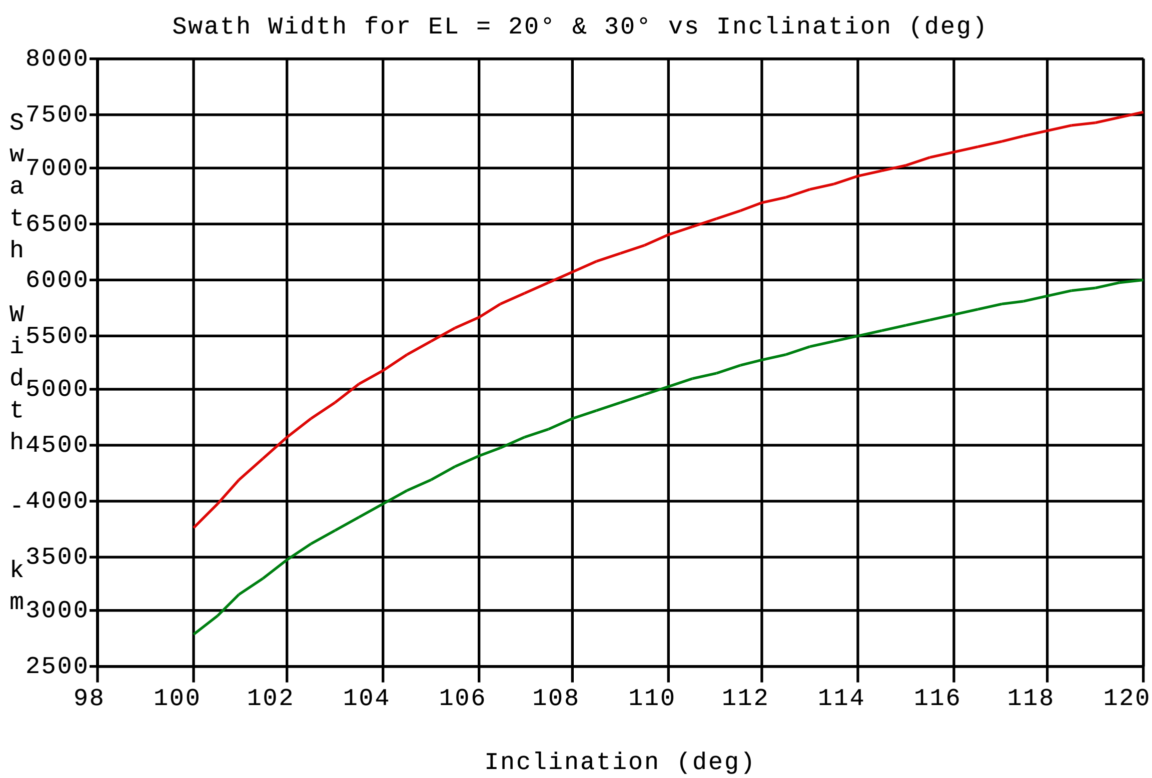
Sun Synchronous Orbit Power Transmission Geometry

Let us assume that a satellite is in a circular Sun synchronous orbit with altitude h and that the radius of the Earth is R_{eq} . Assume that the minimum elevation (angle above the horizon) from which a ground station can receive power is *EL* . We need to find the width of the swath onto which the satellite could possibly beam power, given the altitude h and the minimum elevation, *EL* . We will express the width of the swath both in angular measure (deg) and in linear measure (km).



Using the laws of sines, the angles θ, φ, and ***EL*** can be found as functions of the orbital altitude, h. If the graphs are drawn for the elevation angle taking on the values ***EL*** = 20° and for ***EL*** = 30°, we get the plots of swath width in degrees and in km shown below.


Swath Angle for EL = 20° & 30° vs Inclination (deg)
Swath angle (deg)
70
65
60
55
50
45
40
35
30
25
20
98
100
102
104
106
108
110
112
114
116
118
120
Inclination (deg)


Swath Width for EL = 20° & 30° vs Inclination (deg)
Swath Width-km
8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
98
100
102
104
106
108
110
112
114
116
118
120
Inclination (deg)